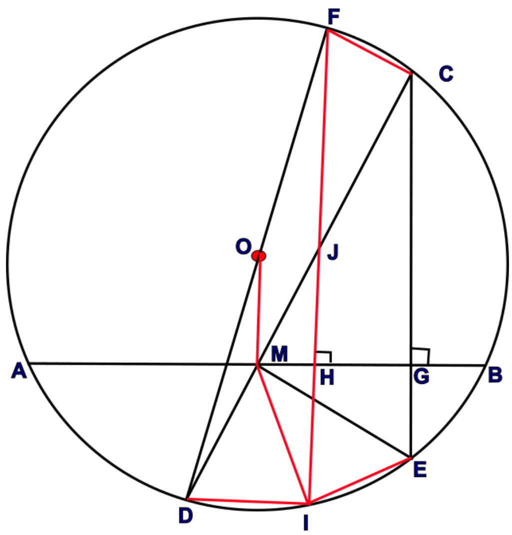
Author's Solution for the question posted on 01.01.2023



Construction:

Draw FH⊥ AB and produce to cut the circle at I. Let FH cut DC at J. Join OM, IM, DI & IE.

Proof :

O is circumcentre and M is the midpoint of chord AB

$\therefore OM \perp AB \,\&\, OM \parallel FJ.$

$\therefore DM = MJ$ ($\because DF$ is diameter, $DO = OF$)

ΔDIJ is a right angled triangle ------------ (1)

$\therefore$ DM = MI = MJ ------------- (2)

$\Rightarrow DM = IM$ ----------------- (3)

FIEC is an isosceles trapezium.

$\therefore FC = IE$ --------------------- (4)

(3) & (4) → $DM^2 + FC^2 = IM^2 + IE^2$ ---------- (5)

(2) → $\angle MIJ = \angle MJI = \angle FJC$ --------------------(6)

$\angle FJC + \angle JFC = 90°$ [$\because \angle FCJ = 90°$, DF is diameter] ------------------- (7)

$\angle JFC = \angle HIE$ [$\because$ $FIEC$ is an isosceles trapezium] --------------(8)

(6), (7) & (8) →

$\angle MIJ + \angle HIE = 90°$ $\therefore \angle MIE = 90°$

$\therefore MI^2 + IE^2 = ME^2$ ----------------------- (9)

(5) & (9) → $DM^2 + FC^2 = EM^2$

$\therefore$ DM, FC & EM can form a right triangle.

Solution by
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